Roll No. | | | | | | | | | | |

**ANNA UNIVERSITY (UNIVERSITY DEPARTMENTS)**

**B.E. /B.Tech / B. Arch (Full Time) - END SEMESTER EXAMINATIONS, NOV/DEC 2024**

(Common to all the Branches)

Semester 1

MA3151 / **MA23C01 MATRICES AND CALCULUS**

(Regulation2023)

Time:3hrs | Max.Marks: 100

| | |
|---|---|
| CO1 | Use the matrix algebra methods for solving practical problems. |
| CO2 | Use differential calculus ideas on several variable functions. |
| CO3 | Apply different methods of integration in solving practical problems by using Beta and Gamma functions. |
| CO4 | Apply multiple integral ideas in solving areas and volumes problems. |
| CO5 | Apply the concept of vectors in solving practical problems. |

**BL – Bloom's Taxonomy Levels**

(L1-Remembering, L2-Understanding, L3-Applying, L4-Analysing, L5-Evaluating, L6-Creating)

**PART- A(10x2=20Marks)**

(Answer all Questions)

| Q.No. | Questions | Marks | CO | BL |
|---|---|---|---|---|
| 1 | If $A = \begin{pmatrix} -1 & 0 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{pmatrix}$ find the eigenvalues of $A^{-1}$. | 2 | 1 | L2 |
| 2 | Write down the index and the nature of the quadratic form $2x^2 + 3y^2 - 7z^2$. | 2 | 1 | L2 |
| 3 | Find the total derivative $du/dt$, if $u = x^2 + y^2, x = at^2, y = 2at$. | 2 | 2 | L2 |
| 4 | Find $\partial^2 u/\partial x \partial y$ if $u = x^y$. | 2 | 2 | L2 |
| 5 | Test whether the improper integral $\int_0^\infty \frac{dx}{1+x^2}$ is convergent or not. | 2 | 3 | L2 |
| 6 | Write down the reduction formula for gamma function. Hence find $\Gamma(5/2)$. | 2 | 3 | L1 |
| 7 | Sketch the region of integration in $\int_0^a \int_{\sqrt{ax}}^a f(x,y)dxdy$. | 2 | 4 | L2 |
| 8 | Evaluate: $\int_0^1 \int_0^1 \int_0^1 xy^2 z dxdydz$. | 2 | 4 | L2 |
| 9 | Find the directional derivative of $\phi = xy^2 + yz^3$ at the point $(2,-1,1)$ in the direction of the vector $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$. | 2 | 5 | L2 |
| 10 | Prove that $\vec{F} = (-x^2 + yz)\hat{\imath} + (4y - z^2x)\hat{\jmath} + (2xz - 4z)\hat{k}$ is solenoidal. | 2 | 5 | L1 |

## PART- B(5x 13=65Marks)

| Q.No. | Questions | Marks | CO | BL |
|---|---|---|---|---|
| 11 (a)(i) | Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$. | 7 | 1 | L3 |
| 11(a) (ii) | Find the characteristic equation of $A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$. Using Cayley Hamiton theorem find $A^{-1}$. (Do not verify Cayley Hamilton theorem.) | 6 | 1 | L3 |
| | **OR** | | | |
| 11 (b) | Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into its canonical form using an orthogonal transformation. | 13 | 1 | L3 |
| | | | | |
| 12 (a)(i) | If $u = \tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. Hence find $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$. | 7 | 2 | L3 |
| 12 (a)(ii) | If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$. | 6 | 2 | L3 |
| | **OR** | | | |
| 12 (b)(i) | The temperature $T$ at any point $(x, y, z)$ in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. | 7 | 2 | L3 |
| 12(b)(ii) | If $x + y + z = u, y + z = uv, z = uvw$, then prove that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$. | 6 | 2 | L3 |
| | | | | |
| 13 (a)(i) | Using the method of differentiation under integral sign, prove that $\int_0^\infty \frac{e^{-x}}{x}(1 - e^{-ax})dx = \log(1 + a)$, where $a > -1$. | 7 | 3 | L4 |
| 13 (a)(ii) | Evaluate the integrals in terms of gamma functions and hence prove that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}}dx \times \int_0^1 \frac{1}{\sqrt{1-x^4}}dx = \pi/4$. | 6 | 3 | L3 |
| | **OR** | | | |
| 13 (b)(i) | Prove that the improper integral $\int_1^\infty \frac{dx}{x^p}$ converges for $p > 1$ and diverges for $p \leq 1$. | 7 | 3 | L4 |

| | | | | |
|---|---|---|---|---|
| 13(b)(ii) | Prove that $\beta(n,n) = \frac{1}{2^{2n-1}}\beta(n,\frac{1}{2})$ and hence prove that $\Gamma(n)\Gamma\left(n+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2n-1}}\Gamma(2n)$. | 6 | 3 | L3 |
| 14 (a)(i) | Find the volume bounded by the cylinder $x^2+y^2=4$, and the planes $y+z=4$, $z=0$. | 7 | 4 | L3 |
| 14 (a) (ii) | By changing the order of integration evaluate $I=\int_0^1\int_0^{1-y} xy\,dxdy$. | 6 | 4 | L4 |
| | **OR** | | | |
| 14 (b)(i) | Find the volume of the tetrahedron bounded by the co-ordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. | 7 | 4 | L3 |
| 14(b)(ii) | By changing into polar co-ordinates evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\,dxdy$. | 6 | 4 | L4 |
| | | | | |
| 15 (a) (i) | State Green's theorem. Using Green's theorem evaluate $$\int_C (x^2+y^2)dx - 2xydy$$ where $C$ is the boundary of the rectangle with vertices $(0,0),(a,0),(a,b),(0,b)$. | 7 | 5 | L3 |
| 15(a)(ii) | Is the vector field $\vec{F}=(2xy)\hat{\imath}+(x^2+2yz)\hat{\jmath}+(y^2+1)\hat{k}$ irrotational? If so, find its scalar potential. | 6 | 5 | L3 |
| | **OR** | | | |
| 15 (b) | Verify Gauss divergence theorem for $\vec{F}=4xz\,\hat{\imath}-y^2\hat{\jmath}+yz\hat{k}$ taken over the cube defined by the planes $x=0, x=1, y=0, y=1, z=0, z=1$. | 13 | 5 | L3 |

## PART- C(1x 15=15Marks)

(Q.No.16 is compulsory)

| Q.No. | Questions | Marks | CO | BL |
|---|---|---|---|---|
| 16.(i) | Diagonalize $A=\begin{pmatrix}1 & 2 & 2\\ 0 & 2 & 1\\ -1 & 2 & 2\end{pmatrix}$, if possible. | 8 | 1 | L5 |
| 16 (ii) | Find the Taylor series expansion of $f(x,y)=x^2y+3y-2$ in powers of $(x-1)$ and $(y-2)$ upto third degree terms. | 7 | 2 | L5 |